PART FOUR

ADDITIONAL INFORMATION

Plate 7.1
RESISTIVITY/PHASE PSEUDOSECTION DATA
Trap Spring Field
Nye Co., Nevada

Line 1
a = 1,250 feet

Explanation of Symbols

Standard Well Symbols

Symbol	Meaning
⊖	Drillhole for which information is unobtainable
○	Drilling in progress at time of map preparation
◇	Shut in
▱	Abandoned
●	Oil well
◇–10,420	Dry hole with total depth indicated
●	Oil well
✳	Gas well
✷	Oil and gas well
○	Water well

Special Well Symbols

Symbol	Meaning
⊙ (2000, 135)	Drilling in progress at the time of the electrical survey; number indicates the amount of drill stem in the hole at the time of data collection
⊙	Well spudded in after completion of the electrical survey
⊙ (0.7)	Number indicates distance of well from the line in terms of a-spacings; all wells within 1.0 a-spacings indicated (pseudosections only)

Culture Symbols

Symbol	Meaning
	Metal pipeline, presumed grounded
	Ungrounded pipeline: non-metal or suspended
	Metal fence
	Electric fence
	Buried telephone or power cable
	Telephone line or standard voltage power line
	Major high voltage power line
	Radio, microwave, or other communications station or tower
	DC pump

Other Symbols

U.S.G.S. standard symbols or as labeled

Zonge Engineering & Research Organization, Inc.

ZONGE ENGINEERING & RESEARCH ORGANIZATION

Apparent Resistivity
Units: ohm-meters
Frequency: 0.125 Hz
Logarithmic contour interval:

10.0
15.8
20.0
25.1
31.6
50.1
63.1
79.4
100.0

Decoupled Phase Angle
Units: milliradians
Frequency: 0.125 Hz
Linear contour interval:

0
5
10
15
20
25
30
40
50

REM Quadrature
Units: normalized imaginary
Frequency: 0.125 Hz
Logarithmic contour interval:

-3981
-2512
-1585
-1000
-631
-398
0
398
631

S 86 W   N 86 E

Topographic profile, 1:1

4800 ft

Plate 7.1
RP Field Data
Trap Spring Field
Line 1

Line 2
Station 23.3

oil–water contact (approx.)
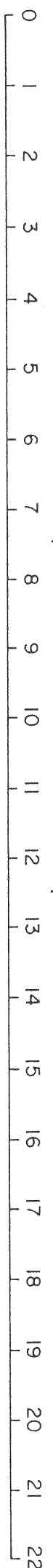
RESISTIVITY/PHASE PSEUDOSECTION DATA
Trap Spring Field
Nye Co., Nevada

Line 2

a = 1,260 feet

Explanation of Symbols

Standard Well Symbols

Symbol	Description
○	Drillhole for which information is unobtainable
○	Drilling in progress at time of map preparation
◇	Shut in
◇-KD	Abandoned
◇	Dry hole with total depth indicated
◇	Oil well
○	Gas well
◇	Oil and gas well
✸	Gas injection well
✷	Water injection well
○	Water well

Culture Symbols

Symbol	Description
•—HG	Metal pipeline; presumed grounded
•——	Ungrounded pipeline: non-metal or suspended
+—+	Electric fence
T—T	Metal fence
⊥	Buried telephone or power cable
⊤	Telephone line or standard voltage power line
⊤⊤	Major high voltage power line
⋏	Radio, microwave, or other communications station or tower

Special Well Symbols

Drilling in progress at the time of the electrical survey; number indicates the amount of drill stem in the hole at the time of data collection

Well spudded in after completion of the electrical survey

Number indicates distance of well from the line in terms of a-spacings; all wells within 1.0 a-spacings indicated (pseudosections only)

Other Symbols

U.S.G.S. standard symbols or as labeled

DC pump

Plate 7.2

ZONGE ENGINEERING & RESEARCH ORGANIZATION

Apparent Resistivity
Units: ohm-meters
Frequency: 0.125 Hz
Logarithmic contour interval:

Decoupled Phase Angle
Units: milliradians
Frequency: 0.125 Hz
Linear contour interval:
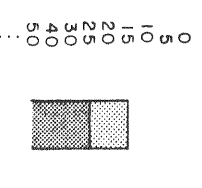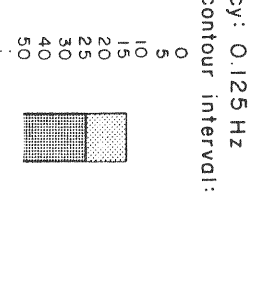
REM Quadrature
Units: normalized imaginary
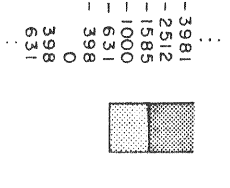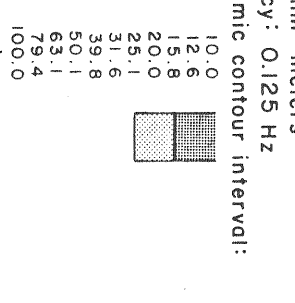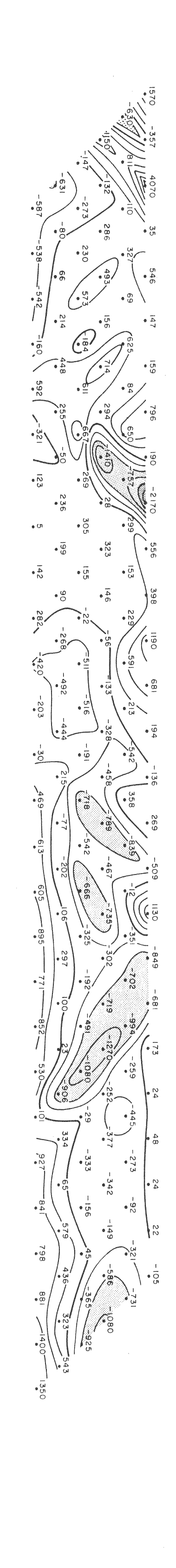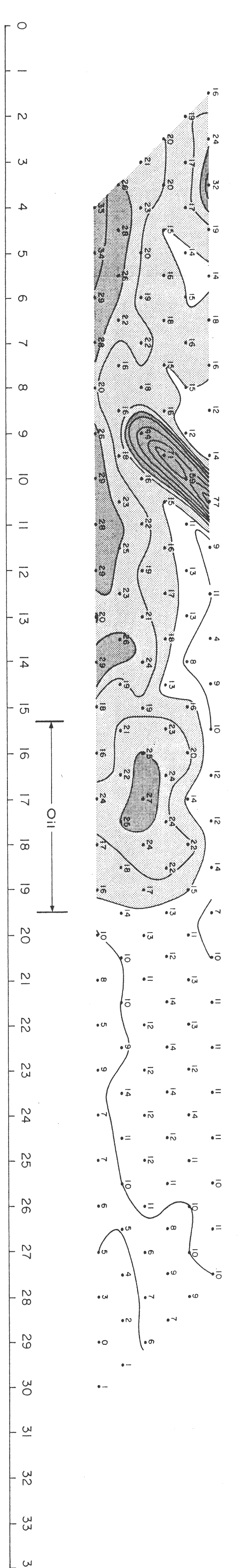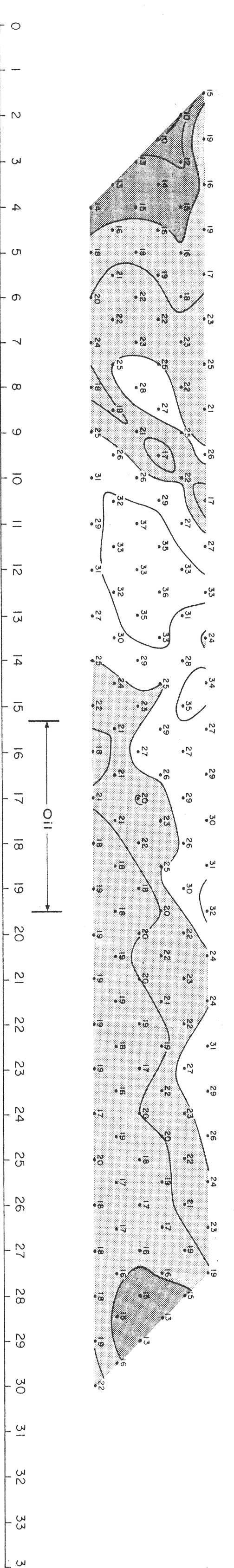Frequency: 0.125 Hz
Logarithmic contour interval:

Plate 7.2
RP Field Data
Trap Spring Field
Line 2

N3E

277
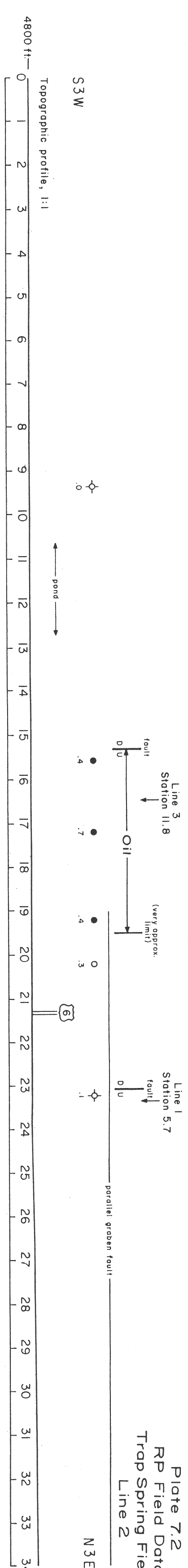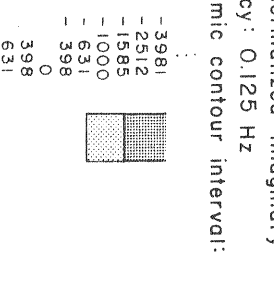
Plate 7.3
RESISTIVITY/PHASE PSEUDOSECTION DATA
Trap Spring Field
Nye Co., Nevada

Line 3
a = 1,250 feet

Standard Well Symbols

- Drillhole for which information is unobtainable
- Drilling in progress at time of map preparation
- Shut in
- Abandoned
- Dry hole with total depth indicated
- Oil well
- Gas well
- Oil and gas well
- Gas injection well
- Water injection well
- Water well

Special Well Symbols

- Drilling in progress at the time of the electrical survey; number indicates the amount of drill stem in the hole at the time of data collection
- Well spudded in after completion of the electrical survey
- Number indicates distance of well from the line in terms of a-spacings; all wells within 1.0 a-spacings indicated (pseudosections only)

Explanation of Symbols

Culture Symbols

- Metal pipeline, presumed grounded
- Ungrounded pipeline: non-metal or suspended
- Metal fence
- Electric fence
- Buried telephone or power cable
- Telephone line or standard voltage power line
- Major high voltage power line
- Radio, microwave, or other communications station or tower

Other Symbols

- DC pump
- U.S.G.S. standard symbols or as labeled

Plate 7.3
RP Field Data
Trap Spring Field
Line 3

ZONGE ENGINEERING & RESEARCH ORGANIZATION

4800 ft.

S 86 W   N 86 E

Topographic profile, 1:1

Line 2
Station 16.4

Apparent Resistivity
Units: ohm – meters
Frequency: 0.125 Hz
Logarithmic contour interval:
12.0
15.8
20.0
25.1
31.6
39.8

Decoupled Phase Angle
Units: milliradians
Frequency: 0.125 Hz
Linear contour interval:
5
10
15
20
25
30
40
50

REM Quadrature
Units: normalized imaginary
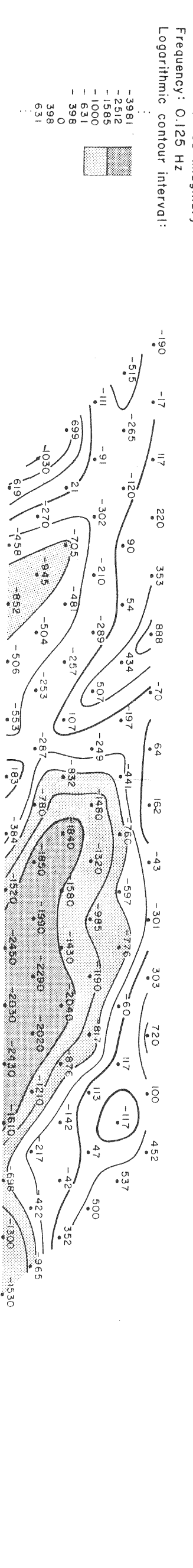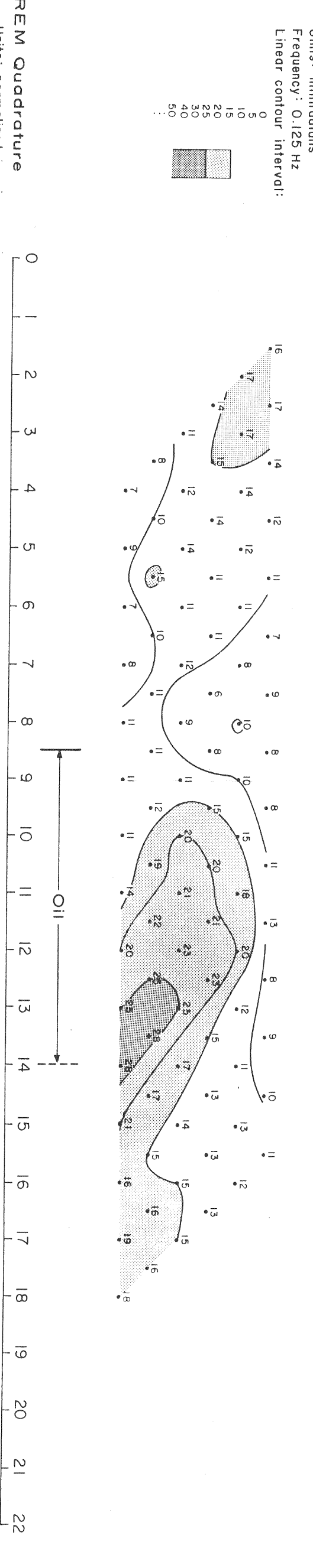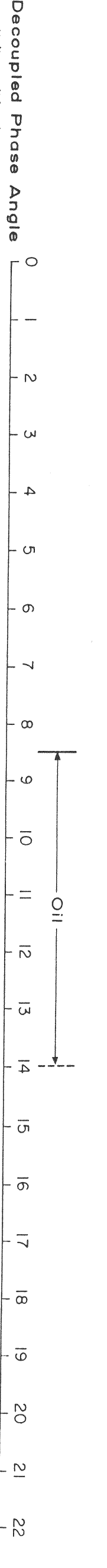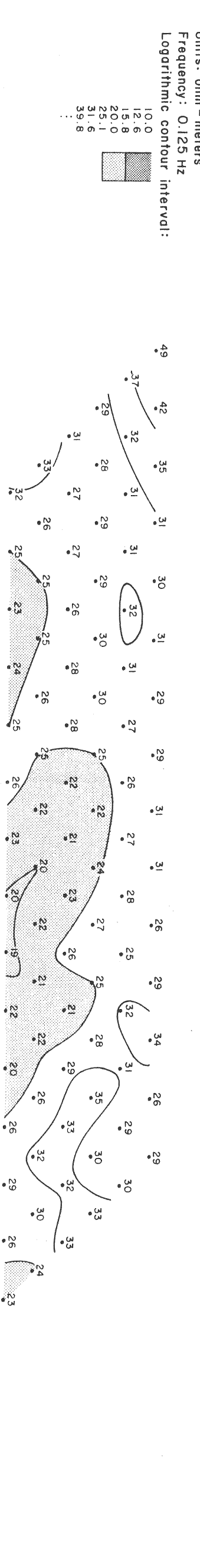Frequency: 0.125 Hz
Logarithmic contour interval:
–398 1
–251 2
–158 5
–100 0
–63 1
–39 8
–0
398
631

Plate 7.3 — Resistivity/Phase Pseudosection Data, Trap Spring Field, Line 3

Chapter 8
Mechanisms of Current Flow in the Earth

8.1 INTRODUCTION

While it is beyond the scope of this volume to provide a full description of current flow in the earth, a brief, practical exposition is provided in order to clarify and explain terms used in the discussion of the case histories. A more detailed, very readable book by Sumner (1976) is recommended should the reader desire more information. The college text by Telford, et al., (1976) is also very helpful and readable. A detailed description of the electrical properties of earth materials and of IP theory may be found in Keller and Frischknecht (1970) and Ward and Fraser (1967).

The earth is, in a geoelectrical sense, an inhomogeneous mess. It is composed of a wide variety of lithologies in a wide variety of geometrical arrangements. It varies in matrix geometry, in pore fluid content, in mineralogy, and so on. Hence it is not surprising that when the earth is stimulated by an electric current, the response to that current is a rather complex one. Sumner (1976) explains why it is helpful to visualize this complex earth as an electric circuit:

> "Although the IP phenomena are basically electrochemical in nature, the systems are really too complex to be represented by a single set of chemical and thermodynamic relationships. The types of ions, their concentrations, and the reactions involved are only generally known. The pore geometry and its relationship with metallic mineral particles are also important. It is quite difficult to represent all these factors explicitly. Thus, it is convenient to use simplified macroscopic analogies in the way of either 'lumped' or 'distributed' electrical circuits."

The equivalent circuit approach is described in detail by Ward and Fraser (1967), Zonge (1972), and Sumner (1976). Equivalent circuits which are used to model the IP response usually consist of resistive and capacitive elements. These circuits attempt to model the frequency dependence of rock resistivities. In actual field measurements, the size and geometry of the measuring system as well as the wire lengths and orientations used enter into the apparent measured results, adding additional frequency and geometric dependence.

In equivalent circuits, the resistor is used to represent the impedance of direct current flow through imperfectly conducting materials. The parameter known as "apparent resistivity" is a measure of the ground resistance, adjusted for the geometry of the measuring points. It is typically calculated for field data at the lowest frequency acquired (0.125 Hz in this volume). Capacitors are used to represent the separation of charge or energy storage in certain ground materials which develops when the ground is subjected to an electric field. This separation of charge, or polarization, will cause a current to flow in the ground after the inducing fields (or currents) are removed, resulting in the "induced polarization" parameter.

8.2
PHYSICAL
MECHANISMS
OF GALVANIC
CURRENT FLOW

Introduction

Few physical properties in nature have as wide a range of values as earth resistivity. As shown in Table 8.1, the resistivity range of common earth materials varies by some eight orders of magnitude, from less than 1 ohm-meter for porous, water-saturated sandstones to 10^6 ohm-meters for dense metamorphic rocks.

To understand this range in resistivites, consider the structure of a typical rock. Current can flow both through the grains which make up the rock and through the pore fluids which surround them. In general, the rock matrix is highly resistive,

TABLE 8.1: RESISTIVITIES OF VARIOUS MATERIALS

ROCKS

Metamorphic	5×10^2 to 10^6 ohm-meters
Igneous	10^3 to 10^6
Clays	1 to 30
Shales	<1 to 600
Sandstone	<1 to 3,000
Porous limestone	100 to 10,000
Dense limestone	$>1,000$
Bulk ground resistivity measured in sedimentary petroleum provinces (typical)	1 to 200

EARTH MINERALS

Chalcopyrite ($Fe_2S_3CU_2S$)	9×10^{-3} to 150
Galena (PbS)	6.8×10^{-6} to 9×10^{-2}
Pyrite (FeS_2)	0.6 to 1.2
Molybdenite (MoS_2)	0.12 to 7.5
Native copper	3×10^{-7} to 1.2
Graphite (current flow parallel to cleavage)	10^{-6} to 36
(current flow perpendicular to cleavage)	9.9×10^{-3} to 28

METALS

Copper, pure	1.6×10^{-8}
Lead	1.9×10^{-7}

unless it contains large amounts of interconnected conductive minerals or clays. On the other hand, typical pore fluids are generally quite conductive, often between 0.1 and 10 ohm-meters. For most rocks near the earth's surface, conduction of electricity is entirely through ground water contained in the pores of the rocks (Keller and Frischknecht, 1970).

There are a number of ways in which galvanic current flow can occur, but in the earth at frequencies less than about 1 kHz, the most important mechanisms are electronic, molecular, ionic, and electrolytic. *Electronic conduction* is accomplished by the shift in energy level of bound electrons. Its contribution is minor in dielectric materials (which make up most earth materials) at all but very high frequencies, but it is an important phenomenon in conductors and semiconductors (such as graphite, some clays, sulfides and some other minerals) even at very low frequencies. *Molecular conduction* involves molecules whose charge distribution is not uniform. When subjected to an AC field, these polarized molecules rotate or oscillate, transferring charge back and forth according to the frequency of the impressed signal. Molecular conduction effects are limited to local charge transfer and are primarily important at frequencies higher than those used in oilfield electrical surveys. *Ionic conduction* is caused by movement of ions within an imperfect crystal lattice under the influence of an external electric field. Its contribution to current flow in saturated, unmineralized rocks is minor, but it is one of the conduction modes in some minerals. *Electrolytic conduction* occurs when free ions are able to flow through a medium due to an applied field. Most electric current flow in the earth occurs through electrolytic conduction, and it is this mechanism which is discussed below.

Characteristics of Electrolytic Conduction

Electrolytic conduction at a given frequency is a function of two factors: the concentration or availability of ions c, and their relative mobilities v. The current flowing through an electrolyte for each volt per meter of electric field applied is expressed by:

$$I = AF\ (c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots) \tag{8.1}$$

where A is cross sectional area through which current flows
F is Faraday's number (96,500 coulombs)
c_n is concentration of species n
and v_n is mobility in meters per second of species n

In solutions with temperatures, pressures, and concentrations typical of most pore fluids, the mobility of a given species does not vary greatly. The important variable is the concentration or availability of the conducting ions.

ION AVAILABILITY

By far, the largest source of ions in earth materials is dissolved salts present in waters which fill the pore spaces in rocks. Up to a point, increased salinity of the electrolytic waters results in a higher availability of ions; likewise the larger the pore space, and hence the larger the volume percent of water, the larger the ion supply. In general, rocks are often water-saturated, and the discussion presented here assumes this to be the case. From equation (8.1) it can be seen that the higher the concentration of an ionic species the more conductive the aqueous solution. Table 8.2 presents resistivities for various concentrations of NaCl at 25°C.

Secondary sources of free ions include escaped cations from covalently and ionically bonded solid-rock materials. Of these, covalently bonded rocks are the larger contributor. Cations such as iron, sodium, and magnesium can be absorbed

TABLE 8.2: RESISTIVITIES OF SALINE SOLUTIONS[1]

Salinity (ppm NaCl at 25°C)	Resistivity (ohm-meters)	Conductivity (millisiemens/meter)
500	10.0	100
1,000	5.0	200
2,700	2.0	500
5,600	1.0	1,000
12,000	0.5	2,000
32,000	0.2	5,000
65,000	0.1	10,000

[1] After Davis and DeWiest (1966).

into broken bonds or substituted for other cations in a crystal lattice in a process called cation exchange. In the process of this exchange, cations are occasionally lost from the face of a crystal lattice into an adjacent electrolyte. Similarly, but to a lesser degree, cations can be lost from ionically bound materials when stray ions of the wrong valence number are bonded into the lattice structure. The weakened lattice gives rise to a statistical probability of cation escape which depends on the bonding energy, the availability of space in which to move, and the lattice temperature.

ION MOBILITY

When an electric field is applied to a rock, cations move toward the negative pole of the field and anions toward the positive pole. Mobility is at first limited by ionic momentum, but the ions then accelerate until their speed is limited by physical or electrical blockages. The time required for an ion to reach its terminal velocity in an aqueous solution is considerably less than a microsecond (Keller and Frisch-knecht, 1970). Physical blockages include increased electrolyte viscosity due to high pressures, low temperatures, large concentrations of ions and other particles, and limitations of pore space. In terms of rock mechanics, pore-space limitations involve porosity (percentage of rock volume involved in pore space), permeability (how the pore spaces are interconnected), and pore-space constriction (width with respect to ion diameter). Pore space constrictions can sort ions by size and reduce their mobilities by crowding, much as traffic is affected by turnpike entryways (Sumner, 1976). Electrical blockages can occur when charge centers such as clay particles attract the mobile ions as they move through the pore spaces. This effect is referred to as membrane polarization.

ARCHIE'S LAW

The general dependence of rock resistivity upon electrolyte salinity and available pore space can be summarized by an empirical relation known as Archie's Law:

$$\rho = a\rho_w \phi^{-m} \tag{8.2}$$

where ρ is the rock resistivity, ρ_w is the water resistivity, and ϕ is the volume percent of porosity of the rock. The values of the coefficients a and m are determined empirically. The coefficient a refers to the type of porosity, and it ranges from values slightly greater than 1 for jointed and fractured rocks to slightly less than 1 for rocks with intergranular and vugular porosity. The coefficient m is called the "cementation factor" and is a measure of the sorting and cementation of the

rock. Rocks which are well-sorted and securely cemented have cementation factors somewhat larger than 2, while poorly sorted or cemented materials have values slightly less than 2. While Archie's Law does not account for secondary effects due to changes in temperature, the presence of metals, electrical blockages, ionization of clay minerals, surface conduction, and other effects, it does serve quite well to relate the more important factors affecting rock resistivity, and it has been widely used in petroleum applications for a number of years. In these applications, the ratio ρ/ρ_w is called the "formation factor" of the rock—a term often used in electrical logging.

ION DIFFUSION AT A SOLID—ELECTROLYTE INTERFACE

There are specific interactions occurring at the interface between a solid material and a conductive electrolyte solution. A solid material will generally possess a small net charge at its edge due to the exposure of ions forming the boundary of a crystal lattice plane (see the discussion of surface conduction below) or a build-up of electrons in a metal. As shown in Figure 8.1, this net charge attracts a layer of free ions of opposite charge, or of dipolar molecules, from the electrolyte at the interface. This layer is called the "fixed layer," since the binding forces involved render it relatively immobile. When there is no applied electric field, the potential drop across the fixed layer is zero.

A second, less dense, "diffuse layer" of ions adjoins the fixed layer and thins out exponentially away from the interface. The potential of the diffuse layer also decreases exponentially away from the surface. The two layers together are referred
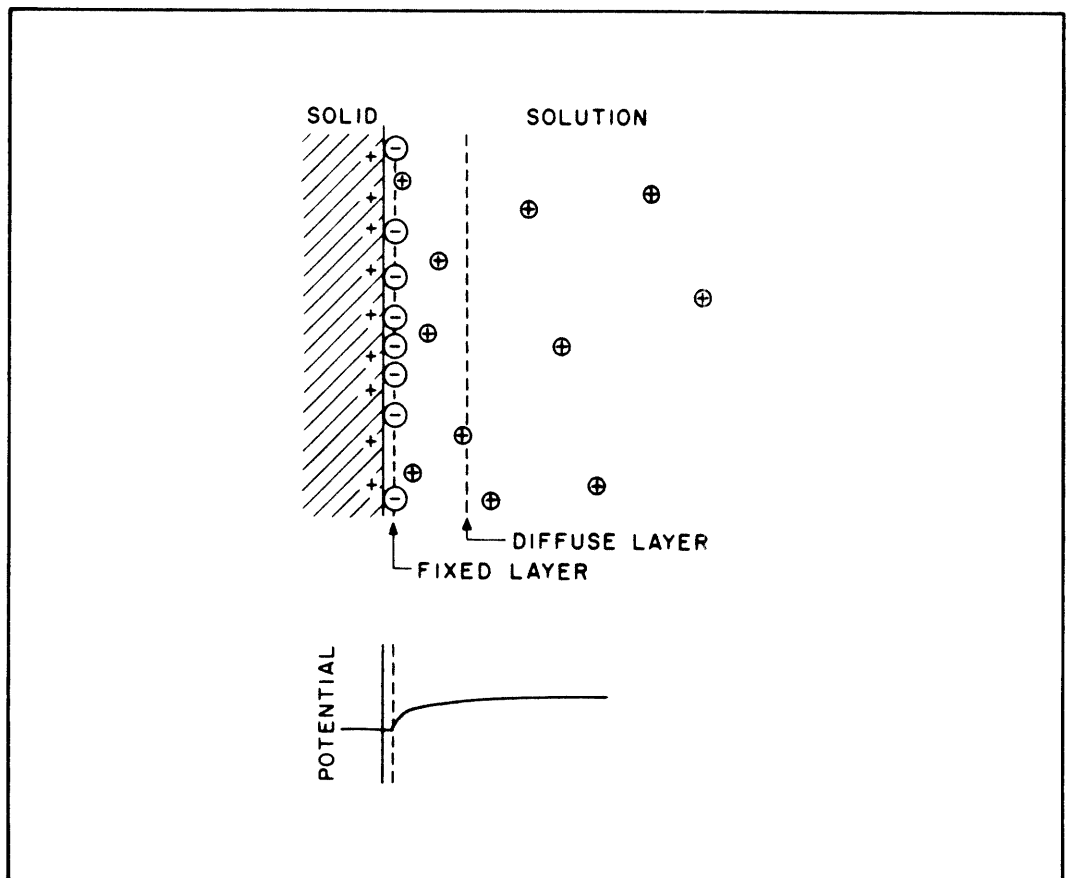

Figure 8.1. Fixed and diffuse (double) layers associated with a solid-electrolyte interface and the resulting Zeta potential (no applied current). After Sumner, 1976 .

to as the "double layer." The potential drop across the double layer is called the "Zeta potential." According to Ward and Fraser (1967), the static width d of the ionic layer across which the Zeta potential occurs depends upon the temperature T of the electrolyte, its dielectric permittivity κ_e, the ionic concentration n, and the ion valence number v_0:

$$d = \left[\frac{\kappa_e kT}{8\pi ne^2 v_0^2} \right]^{1/2} \tag{8.3}$$

in which k is Boltzman's constant
          e is the elementary charge

If an electric field is applied to the double layer, cations will be propelled toward the negative pole of the field and anions toward the positive pole. The net reaction will be a transfer of current as the ion distribution is changed. If the field is due to an alternating current, the ionic distribution will shift back and forth with the reversals of the field polarity, and both the Zeta potential and the distance d will be frequency-dependent.

Electrode Polarization

Electrode polarization generally occurs when an electrolyte adjoins a material which conducts electronically, such as most sulfides, some metallic oxides, and graphite. The magnitude of the electrode polarization depends on the area of semiconductor-electrolyte interface available, the mineral species available, the electrolyte composition, and the pore-fluid content of the rock. Figure 8.2a shows the separation of charge which can develop as ions move in the diffuse layers on opposite sides of a metallic mineral grain which blocks the primary conduction path.
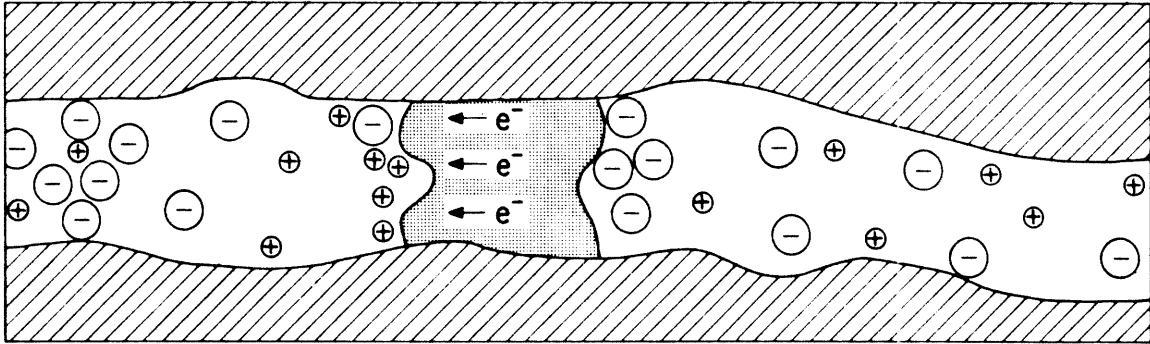
Membrane Polarization

Figure 8.2b illustrates an electrolyte-saturated pore space in a rock in which clay particles are attached along the solid rock walls. The effects of unsatisfied charges in the clay lattice result in an adsorbed cationic cloud around the clay surfaces, forming constrictions in the pore spaces. When an electric field is applied, the relative mobilities of the ions in the pore space become a function of ion size; large ions (usually anions) are blocked by constrictions in the pore spaces, but smaller ions are allowed to pass. This results in zones of ion concentration and ion depletion, or separation of charge, which produces an observable polarization. This effect can also be generated by ion-selective membranes, hence the designation "membrane polarization."

This effect is very difficult to distinguish from electrode polarization, and its contribution to the overall polarizability of a rock can often equal or exceed normal electrode polarization effects. Differences between the two polarization mechanisms can often be determined by making multi-frequency induced polarization measurements.

Membrane effects are very sensitive to specific aspects of rock composition. If existing clays and other materials present a large surface area along the pores, ionic concentration can be high enough that ions cannot be effectively redistributed and significant polarization cannot develop. The length of the pore paths is also important; long pore paths mean slower current response with charge transfer and hence a different response with changing frequency.

Figure 8.2c shows two other sources of polarization. If a metallic particle does not physically block the current flow, two effects can be generated. Charge separation can be induced along the surface of the metallic particle, or charge
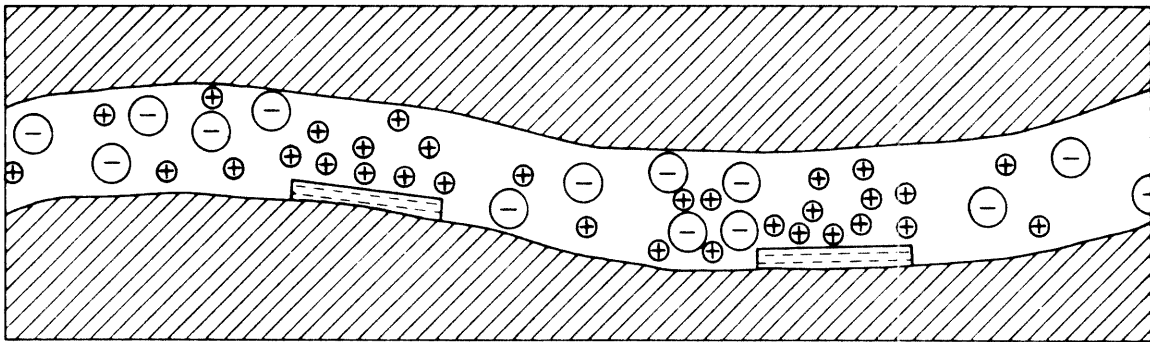
Figure 8.2. Sources of the induced polarization response (after Sauck, 1969, Sumner, 1976, and Scott-Fleming, 1980).

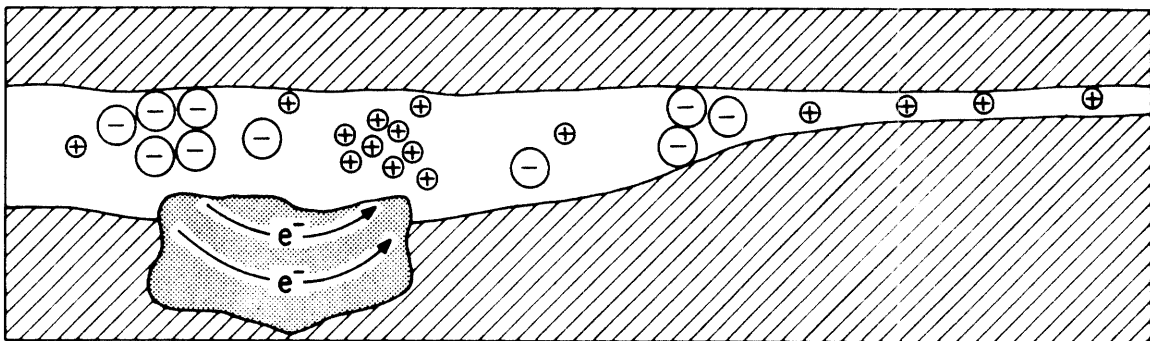build-up associated with electrode polarization may also block ion flow, as in the clay example of Figure 8.2b. These effects will serve to enhance the electrode polarization. Also, if pore spaces narrow sufficiently, large ions can crowd together at the bottleneck permitting charge separation (polarization) to develop as smaller ions continue to flow.

Surface Conduction

Increased ion concentrations associated with double layer adsorption increase the density of charge carriers above what would be normal for an uncharged surface (Figure 8.2b). The result is increased conduction associated with charged surfaces (Ward and Fraser, 1967). Surface conduction is another variation on the ion-diffusion process in which a solid substance presents a net negative charge to an electrolyte.

Clay minerals are particularly noted not only for increasing polarization effects but also for increasing the conductivities of rocks. The importance of surface conduction increases as porosity decreases and as saturation by water decreases (as in the vadose zone or in rocks filled with mixtures of water and resistive fluids such as petroleum liquids). According to Keller and Frischknecht (1970), the critical water saturation appears to be about 25% of total pore space for sandstones and similar permeable rocks, although for less permeable rocks, it may be as high as 70% to 80%. As long as the saturation is above this critical figure, the resistivity of the rock will vary according to the square of the degree of saturation; below the critical figure, the variation is according to the fourth or fifth power of the degree of saturation, and resistivities climb rapidly.

The two main polarization effects, electrode polarization and membrane polarization, only occur under the influence of an electric field. They are also frequency-dependent. The decoupled phase angles presented in the case histories are thought to represent the sum of all membrane and electrode polarization effects, with all electromagnetic effects (see below and Chapter 9) removed. It has been found (Zonge, 1972) that the frequency dependence of the IP response can be diagnostic for a few economic and noneconomic minerals and, with proper care, spectral measurements can be used to distinguish these minerals, even in the field.

8.3
THE ORIGIN OF
INDUCTIVE
COUPLING

In acquiring field data, the apparent resistivity or induced polarization (IP) measurements are contaminated by the effects of the actual measuring system and its interaction with the ground. Normally a four-electrode system is utilized, with a pair of electrodes (or a dipole) used for transmitting current into the ground, and the other pair used for detecting the resultant voltage in the ground. The wires connecting the electrodes in these measurements will couple together inductively, and depending upon their relative orientation will respond either inductively or capacitively, i.e., either subtracting from or adding to the induced polarization response. This response is usually referred to as inductive electromagnetic (EM) coupling and is the total EM coupling response when operating over a homogeneous half-space. However, when the earth is layered or otherwise inhomogeneous, a second frequency-dependent EM coupling parameter is generated which is a function of the actual position of the current (transmitter) and voltage (receiver) electrodes, as well as the orientation of the wires connecting them. Depending upon the geometry of the measuring array and the relative resistivities of the layers in the earth, this EM parameter will either add to or subtract from the combined IP and inductive

EM responses described above. It is this latter EM coupling effect that we try to measure for the residual electromagnetic (REM) parameter.

It should be noted here that the combined EM effect (inductive plus geometric coupling) can often exceed the actual ground IP response by a factor of 10 or more, and can change the apparent ground resistivity by as much as 50 percent, for a typical oilfield survey with large electrode separations. It is therefore very important that any resistivity and IP data gathered in this environment be corrected for EM effects, and this can only be done by making multiple frequency measurements.

Chapter 9 contains a more complete discussion of the principles of EM theory as applied to hydrocarbon exploration.

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Zonge, K.L., 1972, Electrical properties of rocks as applied to geophysical prospecting: PhD dissertation, University of Arizona, Tucson.

"Engineering Procedure," from an unknown but astute source

Every new engineer must learn early that it is never good taste to designate the sum of two quantities in the form:

$$1 + 1 = 2 \tag{1}$$

Anyone who has made a study of advanced mathematics is aware that $1 = \ln e$ and that $1 = \sin^2 x + \cos^2 x$; further,

$$2 = \sum_{n=0}^{\infty} 1/2^n$$

Therefore, equation (1) can be expressed more scientifically as:

$$\ln e + (\sin^2 x + \cos^2 x) = \sum_{n=0}^{\infty} 1/2^n \tag{2}$$

This may be further simplified by use of the relations: $1 = \cosh y \sqrt{1 - \tanh^2 y}$ and $e = \lim_{z \to \infty} (1 + 1/z)^z$. Equation (2) may therefore be rewritten

$$\ln \left[\lim_{z \to \infty} (1 + 1/z)^z \right] + (\sin^2 x + \cos^2 x) = \sum_{n=0}^{\infty} \frac{\cosh y}{2^n} \sqrt{1 - \tanh^2 y} \tag{3}$$

At this point, it should be obvious that equation (3) is much clearer and more easily understood than equation (1). Other methods of a similar nature could be used to clarify equation (1) but these are easily discovered once the reader grasps the underlying principles.

We hope this chapter will form the basis of a reverse solution.

Chapter 9
Principles of Electromagnetic Theory as Applied to Petroleum Exploration

**9.1
INTRODUCTION**

Electrical exploration for oil and gas has all too frequently been pursued by persons with very little understanding of how electromagnetic energy propagates in the earth. As a result, some rather outrageous claims have been made by some practitioners of electrical techniques. The petroleum industry has responded with justifiable skepticism, and electrical techniques have drifted in and out of favor several times since they were first applied to oil and gas exploration in the early 1900s.

However, the validity of these techniques is not in question, only the ways in which they are sometimes used. It is therefore beneficial to highlight the essential principles of electromagnetic theory as applied to petroleum exploration, in the hope that future efforts will be more firmly tied to reality.

This chapter provides the basic theory pertaining to induced polarization measurements, with some references to controlled source audiofrequency magneto-telluric measurements. We shall consider the behavior of the ground as it is pulsed with an alternating current from a grounded dipole source. The discussion presents the basic defining relations, develops a vector wave equation involving the potential fields, then proceeds to outline the solution to the wave equation for certain situations of special interest in petroleum exploration. The treatment roughly follows that of Ward (1967) and Sunde (1967), with some references to Keller and Frischknecht (1966). Table 9.1 lists the quantities, units, and constants used in this chapter. Figure 9.1 shows the coordinate systems used.

**9.2
MAXWELL'S
EQUATIONS**

If an alternating current is caused to flow in a conductive medium like the earth, an electromagnetic field is established. In the formulation which follows, four parameters may be used to describe this field:

$\vec{E}$ Electric field intensity
$\vec{D}$ Electric flux density
$\vec{H}$ Magnetic field intensity
$\vec{B}$ Magnetic flux density

TABLE 9.1: SUMMARY OF FIELD QUANTITIES AND UNITS

Units

Unit	Symbol	Meaning	Equivalent Units
ohm	ohm	Electrical resistance	v/amp
ohm-meter	ohm-m	Electrical resistivity	v-m/amp
mhos/meter	mhos/m	Electrical conductivity	amp/v-m
ampere	amp	Electrical current	v/ohm
volt	v	Voltage	amp-ohm
weber	wb	Magnetic flux	v-sec
coulomb	coul	Electric charge	amp-sec
henry	hr	Magnetic inductance	ohm-sec
farad	fd	Capacitance	sec/ohm

Field Quantities

$\vec{E}$	Electric field intensity (v/m)
$\vec{D}$	Electric flux density (coul/m^2)
$\vec{H}$	Magnetic field intensity (amp/m)
$\vec{B}$	Magnetic flux density (wb/m^2)
$\vec{J}$	Electric current density (amp/m^2)
q_v	Electric charge density (coul/m^3)
$\vec{A}^*$	Electric vector potential (coul/m)
$\vec{A}$	Magnetic vector potential (wb/m)
ϕ	Electric scalar potential (v)
ϕ^*	Magnetic scalar potential (amp)
$\vec{\pi}$	Electric Hertz vector potential (v-m)
$\vec{\pi}^*$	Magnetic Hertz vector potential (amp-m)
$\vec{P}_v$	Electric dipole moment per unit volume (coul/m^2)
$\vec{M}_v$	Magnetic dipole moment per unit volume (amp/m)
V	Voltage drop (v)
I	Electric current (amp)
Z	Complex impedance (ohm)
R_{TE}	Transverse electric coefficient (unitless)
R_{TM}	Transverse magnetic coefficient (unitless)
Q	"Q" coupling function (ohm)
P	"P" coupling function (ohm)

Media Properties

σ	Electrical conductivity (mhos/m)
ρ	Electrical resistivity (ohm-m)
ϵ	Electric permittivity (fd/m)
μ	Magnetic permeability (hr/m)

Media Properties (continued)

κ_e	Relative permittivity (dielectric constant) (unitless)
κ_e	Relative permeability (unitless)
χ_e	Electric susceptibility (unitless)
χ_m	Magnetic susceptibility (unitless)
$\vec{P}_e$	Electric polarization vector (coul/m^2)
$\vec{P}_m$	Magnetic polarization vector (amp/m)
k	Propagation constant (1/m)
α	Phase constant (1/m)
β	Attenuation constant (1/m)
δ	Skin depth (m)
v_p	Phase velocity (m/sec)
λ	Wavelength (m)

Other Quantities

t	Time (sec)
ω	Angular frequency (rad/sec)
f	Linear frequency (Hz or cycles/sec)
r	Distance (m)
a	Dipole size in dipole-dipole array (m)
n	Dipole separation (in terms of "a") in dipole-dipole array (m)

Constants

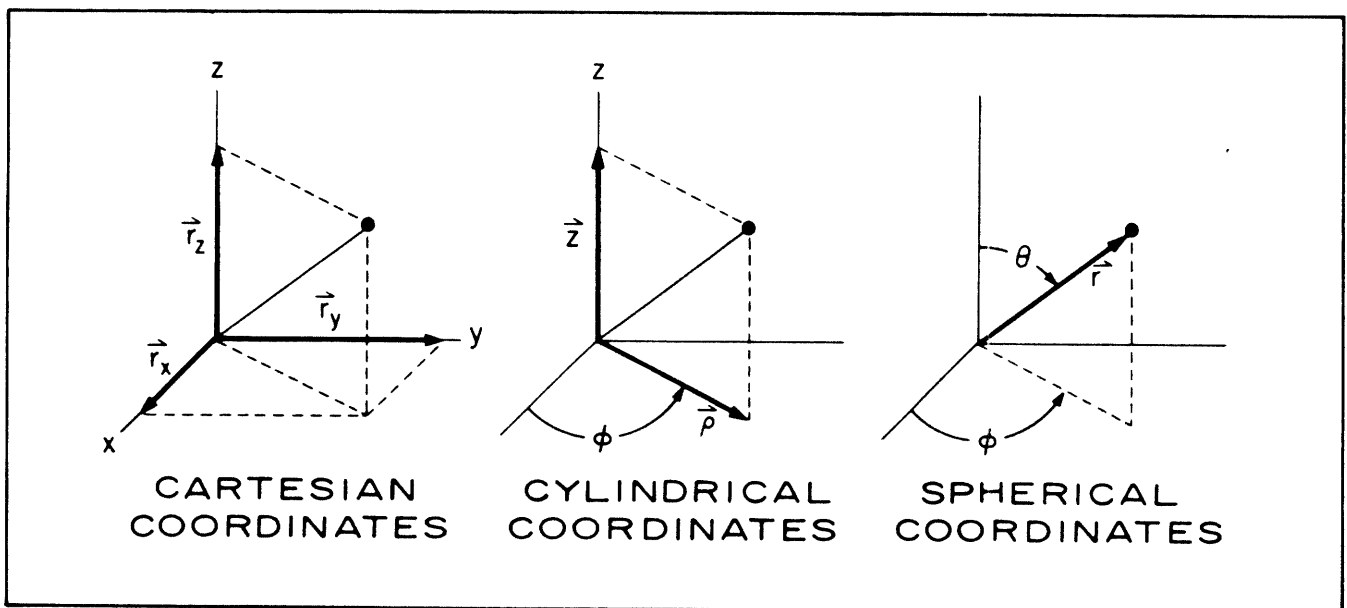|		
ϵ_o	Free space electric permittivity	8.854×10^{-12} fd/m
μ_o	Free space magnetic permeability	$4\pi \times 10^{-7}$ hr/m
c	Speed of light in a vacuum	3.00×10^8 m/sec

Figure 9.1. Cartesian, cylindrical, and spherical coordinate systems.

These parameters are interrelated by four fundamental laws of electromagnetism, known as Maxwell's equations:

$$\vec{\nabla} \times \vec{E} + \partial \vec{B}/\partial t = 0 \qquad \text{(Faraday's Law)} \qquad (9.1)$$

$$\vec{\nabla} \times \vec{H} - \partial \vec{D}/\partial t = \vec{J} \qquad \text{(Ampere's Law)} \qquad (9.2)$$

$$\vec{\nabla} \cdot \vec{D} = q_v \qquad \text{(Coulomb's Law)} \qquad (9.3)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(Continuous flux law)} \qquad (9.4)$$

In these, $\vec{J}$ is the electric current density and q_v is the electric charge density. These relations are largely the result of experimental work in electricity and magnetism conducted during the 19th century. First assembled as a group by J.C. Maxwell in 1863, they form a general and elegant description of electromagnetic phenomena. A direct result of these equations, the so-called equation of continuity or conservation of electric charge, relates current density to charge density:

$$\vec{\nabla} \cdot \vec{J} + \partial q_v/\partial t = 0 \qquad (9.5)$$

9.3 PHYSICAL PROPERTIES OF A CONDUCTIVE MEDIUM

Materials in the earth may normally be described by three physical properties: conductivity (σ), electric permittivity (ϵ), and magnetic permeability (μ). Conductivity is a measurement of the current flow characteristics of the ground. Materials with a high conductivity are known as conductors; materials with a low conductivity are known as insulators or dielectrics. The inverse of conductivity, resistivity (ρ), is the quantity normally measured in Zonge Engineering hydrocarbon surveys:

$$\rho = 1/\sigma \qquad (9.6)$$

Resistivity is the resistance to electrical current flowing through a cross-sectional area, measured as a function of the distance over which the current flows; hence, the units for resistivity are ohm-m^2/m, or ohm-m. As will be seen later, resistivity is also a function of the geometry of the array used to measure it.

Two additional field quantities, electric permittivity and magnetic permeability, are defined by empirically-derived relationships. For free space, the permittivity (ϵ_o) and permeability (μ_o) are:

$$\epsilon_o = 8.854 \times 10^{-12} \text{ fd/m}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ hr/m}$$

For a given material, the ratios

$$\kappa_e = \epsilon/\epsilon_o \qquad (9.7)$$

and

$$\kappa_m = \mu/\mu_o \qquad (9.8)$$

are known as the relative permittivity (or dielectric constant) and the relative permeability, respectively. The departures of these quantities from unity,

$$\chi_e = \kappa_e - 1 \qquad (9.9)$$

$$\chi_m = \kappa_m - 1 \qquad (9.10)$$

are called the electric and magnetic susceptibilities, respectively.

While the dielectric constant is close to unity for most earth materials found in oilfield environments, the magnetic permeability shows a greater range of values, depending upon the amounts of magnetic minerals contained in the rocks. Most sediments are nonmagnetic ($\mu = \mu_o$), but occasionally one encounters diamagnetic materials, which have permeabilities very slightly lower than the permeability of free space ($\mu < \mu_o$), or paramagnetic materials, which have permeabilities very slightly higher than the permeability of free space ($\mu > \mu_o$). Materials which have very high permeabilities are called ferromagnetic. The most common ferromagnetic material found in the earth is magnetite.

In the case of an electromagnetic field propagating through a simple, isotropic material, the basic field quantities can be related to each other in terms of the physical properties of the material:

$$\vec{D} = \epsilon \vec{E} \tag{9.11}$$

$$\vec{B} = \mu \vec{H} \tag{9.12}$$

The electric polarization vector $\vec{P}_e$ and the magnetic polarization vector $\vec{P}_m$ can be defined as:

$$\vec{P}_e = \vec{D} - \epsilon_o \vec{E} = \chi_e \epsilon_o \vec{E} \tag{9.13}$$

$$\vec{P}_m = 1/\mu_o \vec{B} - \vec{H} = \chi_m \vec{H} \tag{9.14}$$

These quantities describe the atomic dipolar polarization due to an impressed field.

An important relation between current density and electric field intensity is an expression of Ohm's Law:

$$\vec{J} = \sigma \vec{E} \tag{9.15}$$

This equation will be used later on to derive the necessary relations for calculating resistivity for induced polarization surveys.

9.4 DEVELOPMENT OF THE VECTOR WAVE EQUATION

An electromagnetic wave propagating through a material can be described in terms of potential functions. In this development, the potential functions are derived in a mathematically convenient manner. While this treatment may seem somewhat arbitrary at first, the potential functions can be used to enhance our appreciation of the physical principles involved.

According to the continuous flux law of equation (9.4), the flux of a magnetic field going into a closed surface is the same as the flux going out:

$$\vec{\nabla} \cdot \vec{B} = 0$$

This suggests that $\vec{B}$ is the curl of some vector $\vec{A}$, since the divergence of the curl is zero:

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{9.16}$$

From equation (9.1), we have

$$\vec{\nabla} \times (\vec{E} + \partial\vec{A}/\partial t) = 0 \tag{9.17}$$

This insures that the vector $(\vec{E} + \partial\vec{A}/\partial t)$ is irrotational and is therefore the gradient of some scalar function ϕ:

$$\vec{E} + \partial\vec{A}/\partial t = -\vec{\nabla}\phi$$

or

$$\vec{E} = -\vec{\nabla}\phi - \partial\vec{A}/\partial t \tag{9.18}$$

The vector $\vec{A}$ is called the magnetic vector potential, and ϕ is the electric scalar potential. Note that in the case of a time-invariant (static) electric field, equation (9.18) reduces to

$$\vec{E} = -\vec{\nabla}\phi \tag{9.19}$$

which is a familiar result of static electromagnetic theory.

We wish to write two linear differential equations, one for $\vec{A}$ and one for ϕ. Taking equation (9.2) and substituting from equations (9.11), (9.12), (9.16), and (9.18), we obtain

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} + \mu\epsilon\,\vec{\nabla}\,\partial\phi/\partial t + \mu\epsilon\,\partial^2\vec{A}/\partial t^2 = \mu\vec{J} \tag{9.20}$$

$$\nabla^2\,\phi + \vec{\nabla}\cdot\partial\vec{A}/\partial t = -q_v/\epsilon \tag{9.21}$$

Contained implicitly in the term for current density $\vec{J}$ are two parts—a primary source $\vec{J}_p$ and a secondary part $\vec{J}_s$. The secondary part has a contribution of

$$\mu\sigma\,(-\vec{\nabla}\phi - \partial\vec{A}/\partial t)$$

which can be explicitly included in equation (9.20) such that

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} + \mu\epsilon\,\vec{\nabla}\,\partial\phi/\partial t + \mu\epsilon\,\partial^2\vec{A}/\partial t^2 + \mu\sigma\vec{\nabla}\phi + \mu\sigma\,\partial\vec{A}/\partial t = \mu\vec{J}_p \tag{9.22}$$

in which $\vec{J}_p = \sigma\vec{E}_p$ and $\vec{J}_s = \sigma\vec{E}_s$.

A vector identity in cartesian coordinates provides that

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}\,\vec{\nabla}\cdot\vec{A} - \vec{\nabla}\cdot\vec{\nabla}\vec{A} \tag{9.23}$$

Further, since the vector $\vec{A}$ is arbitrarily obtained, we can impose the Lorentz condition:

$$\vec{\nabla}\cdot\vec{A} + \mu\epsilon\,\partial\phi/\partial t + \mu\sigma\phi = 0 \tag{9.24}$$

Using equations (9.23) and (9.24), we can rewrite equations (9.22) and (9.21) as

$$\nabla^2\,\vec{A} - \mu\epsilon\,\partial^2\vec{A}/\partial t^2 - \mu\sigma\,\partial\vec{A}/\partial t = \mu\vec{J}_p \tag{9.25}$$

$$\nabla^2\,\phi - \mu\epsilon\,\partial^2\phi/\partial t^2 - \mu\sigma\,\partial\phi/\partial t = -q_v/\epsilon \tag{9.26}$$

The solutions to these two inhomogeneous wave equations can be derived in terms of $\vec{A}$ and ϕ, but it is more convenient to introduce a third potential term, $\vec{\pi}$, which is called the electric Hertz vector potential. This potential has the advantage of combining $\vec{A}$ and ϕ into a single wave equation. The Hertz vector is defined by

$$\vec{A} = \mu\epsilon\,\partial\vec{\pi}/\partial t + \mu\sigma\vec{\pi} \tag{9.27}$$

$$\phi = -\vec{\nabla}\cdot\vec{\pi} \tag{9.28}$$

This allows equation (9.25) to be rewritten as

$$\mu\sigma \ [\nabla^2 \vec{\pi} - \mu\epsilon \ \partial^2\vec{\pi}/\partial t^2 - \mu\sigma \ \partial\vec{\pi}/\partial t] + \mu\epsilon \ \partial/\partial t \ [\nabla^2\vec{\pi} - \mu\epsilon \ \partial^2\vec{\pi}/\partial t^2 - \mu\sigma \ \partial\vec{\pi}/\partial t] = \mu \ \partial\vec{P_v}/\partial t \quad (9.29)$$

in which $\vec{P_v}$ is the dipole moment per unit volume, defined by

$$\vec{J}_p = \partial\vec{P_v}/\partial t \quad (9.30)$$

The form of equation (9.29) can be recognized more easily by representing the bracketed terms by "$\vec{X}$", in which case we have

$$\vec{X} + \epsilon/\sigma \ \partial\vec{X}/\partial t = - \ 1/\sigma \ \partial\vec{P_v}/\partial t \quad (9.31)$$

The solution to this inhomogeneous differential equation may be obtained by considering two limiting cases. The first limiting case is for a source-free region with transient fields ($\vec{P_v} = 0$, $\sigma \neq 0$). Equation (9.31) then reduces to a homogeneous differential equation:

$$\vec{X} + \epsilon/\sigma \ \partial\vec{X}/\partial t = 0 \quad (9.32)$$

The solutions are of the form

$$\vec{X} = e^{-(\sigma/\epsilon)t} \quad (9.33)$$

The σ/ϵ term is a measure of the decay time of transients due to the induced field. In general, this time is negligibly small (less than 10^{-6} second), in which case we have

$$\vec{X} = 0 \quad (9.34)$$

Following the pattern of equations (9.25) and (9.26), this results in:

$$\nabla^2\vec{\pi} - \mu\epsilon \ \partial^2\vec{\pi}/\partial t^2 - \mu\sigma \ \partial\vec{\pi}/\partial t = 0 \quad (9.35)$$

The opposite limiting case is for a region with sources but no transients ($\vec{P_v} \neq 0$, $\sigma = 0$), in which case we have

$$\partial\vec{X}/\partial t = - \ 1/\epsilon \ \partial\vec{P_v}/\partial t \quad (9.36)$$

or

$$\vec{X} = -(1/\epsilon) \ \partial\vec{P_v}/\partial t + \vec{C} \quad (9.37)$$

Neglecting the constant of integration, this can be rewritten as:

$$\nabla^2\vec{\pi} - \mu\epsilon \ \partial^2\vec{\pi}/\partial t^2 = - \ \vec{P_v}/\epsilon \quad (9.38)$$

Since the total field is the sum of the primary field and the secondary field, we can sum equations (9.34) and (9.38) to obtain

$$\vec{X} = 0 + (-\vec{P_v}/\epsilon) \quad (9.39)$$

which converts back to our previous nomenclature as:

$$\nabla^2\vec{\pi} - \mu\epsilon \ \partial^2\vec{\pi}/\partial t^2 - \mu\sigma \ \partial\vec{\pi}/\partial t = - \ \vec{P_v}/\epsilon \quad (9.40)$$

This has the same form of equations (9.25) and (9.26). The electric and magnetic fields can be obtained in terms of $\vec{\pi}$ by equations (9.19) and (9.16):

$$\vec{E} = \vec{\nabla}\ \vec{\nabla} \cdot \vec{\pi} - \mu\epsilon\ \partial^2\vec{\pi}/\partial t^2 - \mu\sigma\ \partial\vec{\pi}/\partial t \tag{9.41}$$

$$\vec{B} = \mu\epsilon\ \vec{\nabla} \times\ \partial\vec{\pi}/\partial t + \mu\sigma\ \vec{\nabla} \times \vec{\pi} \tag{9.42}$$

So far we have dealt with the electric Hertz vector only, in which magnetic sources and transients are not considered. To generalize the discussion, we introduce the electric vector potential, $\vec{A}^*$, and the magnetic scalar potential, ϕ^*. Following our earlier development, equations (9.25), (9.26), (9.27), and (9.28) can be re-written as follows:

$$\nabla^2 \vec{A}^* - \mu\epsilon\ \partial^2\vec{A}^*/\partial t^2 - \mu\sigma\ \partial\vec{A}^*\partial t = -\ \mu\epsilon\ \partial\vec{M}_v/\partial t \tag{9.43}$$

$$\nabla^2\ \phi^* - \mu\epsilon\ \partial^2\phi^*/\partial t^2 - \mu\sigma\ \partial\phi^*/\partial t = \vec{\nabla} \cdot \vec{M}_v \tag{9.44}$$

$$\vec{A}^* = \mu\epsilon\ \partial\vec{\pi}^*/\partial t + \mu\sigma\vec{\pi}^* \tag{9.45}$$

$$\phi^* = -\ \vec{\nabla} \cdot \vec{\pi}^* \tag{9.46}$$

in which $\vec{M}_v$ is the magnetic dipole moment per unit volume. We can therefore write for the magnetic Hertz vector:

$$\nabla^2\ \vec{\pi}^* - \mu\epsilon\ \partial^2\vec{\pi}^*/\partial t^2 - \mu\sigma\ \partial\vec{\pi}^*/\partial t = -\ \vec{M}_v \tag{9.47}$$

This is the magnetic analog of equation (9.40). Finally, considering both electric and magnetic sources with transients, we can write the electric and magnetic fields as follows:

$$\vec{E} = \nabla^2\vec{\pi} - \mu\epsilon\ \partial^2\vec{\pi}\ /\ \partial t^2 - \mu\sigma\ \partial\vec{\pi}/\partial t - \mu\ \vec{\nabla} \times\ \partial\vec{\pi}^*/\partial t \tag{9.48}$$

$$\vec{H} = \nabla^2\vec{\pi}^* - \mu\epsilon\ \partial^2\vec{\pi}^*/\partial t^2 - \mu\sigma\ \partial\vec{\pi}^*/\partial t + \epsilon\ \vec{\nabla} \times\ \partial\vec{\pi}/\partial t + \sigma\ \vec{\nabla} \times \vec{\pi} \tag{9.49}$$

**9.5
BOUNDARY
CONDITIONS**

Solving the potential equations in a layered earth requires that we say something about the behavior of $\vec{E}$ and $\vec{H}$ as they traverse the boundary between two media with properties μ_1, ϵ_1, σ_1 and μ_2, ϵ_2, σ_2 (Figure 9.2). The following boundary conditions can be obtained from elementary calculus and from the concepts just developed:

- The normal component of $\vec{D}$ is continuous across the interface $(D_{2N} - D_{1N} = 0)$.
- The normal component of $\vec{B}$ is continuous $(B_{2N} - B_{1N} = 0)$.
- The tangential component of $\vec{E}$ is continuous $(E_{2T} - E_{1T} = 0)$.
- The tangential component of $\vec{H}$ is continuous $(H_{2T} - H_{1T} = 0)$.
- Current density is continuous $(J_{1N} = J_{2N})$.
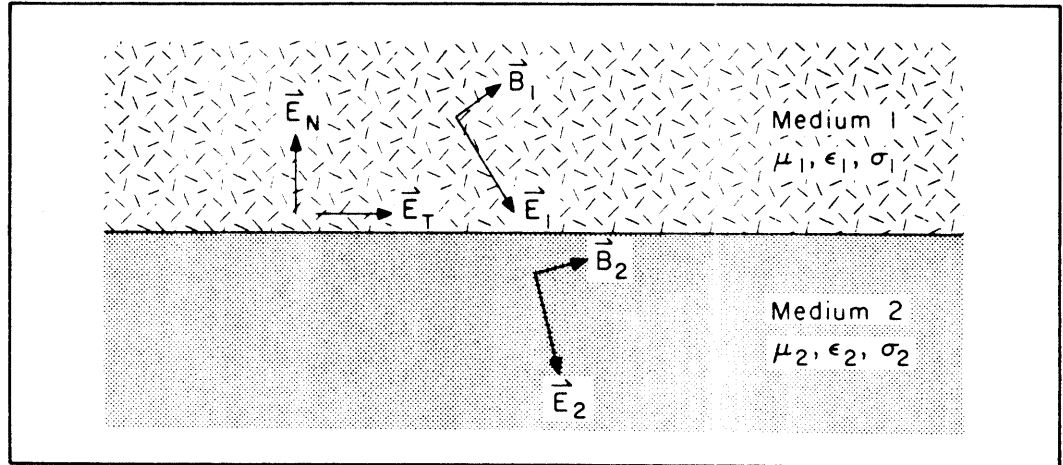- Potentials are continuous if no sources are present $(\phi_1 = \phi_2, \phi_1^* = \phi_2^*)$.

Figure 9.2. Boundary conditions at the interface of two media.

9.6 DC SOLUTIONS TO THE VECTOR WAVE EQUATION

Introduction

Before advancing to the more general AC solutions to the vector wave equations, it is useful to discuss the much simpler DC solutions. We recall from equation (9.18) that

$$\vec{E} = -\vec{\nabla}\phi - \partial\vec{A}/\partial t$$

For the time-invariant DC case, this reduces to equation (9.19):

$$\vec{E} = -\vec{\nabla}\phi$$

The scalar ϕ is the potential at a point which is at a distance r from a single point source. The potential is referenced to infinity:

$$\phi = -\int_r^\infty E_r dr \tag{9.50}$$

From equation (9.15), using resistivity in place of conductivity ($\rho = 1/\sigma$),

$$E_r = J_r \rho \tag{9.51}$$

This can be substituted into equation (9.50) to obtain, for a homogeneous whole-space,

$$\phi_r = -\int_r^\infty E_r dr = I\rho/4\pi r \tag{9.52}$$

The solutions of interest in this chapter involve potentials at points on the surface of the earth, which means we will seek equations for a homogeneous half-space (the earth) bounded by a perfect insulator (the air). In this case, the current density is twice that of a whole-space, and the potential becomes

$$\phi_r = -\int_r^\infty E_r dr = I\rho/2\pi r \tag{9.53}$$

Homogeneous
Earth
Resistivity

All induced polarization methods commonly used in hydrocarbon explor-
ation involve the use of four grounded electrodes. Two electrodes are used for the
current or transmitting dipole, and two are used for the receiving dipole, which
measures the ground potential. Figure 9.3 shows a general electrode arrangement for
a ground survey over a homogeneous earth. Current is transmitted into the dipole
MN; it enters (+ I) electrode M and exits (− I) electrode N. The ground potentials
measured at electrodes A and B, referenced to the source electrodes M and N, may
be written as follows:

$$\phi_{AM} = I\rho/2\pi r_{AM}$$

$$\phi_{AN} = I\rho/2\pi r_{AN}$$

$$\phi_{BM} = I\rho/2\pi r_{BM}$$

$$\phi_{BN} = I\rho/2\pi r_{BN}$$

By superposition, the potential difference across the receiving dipole AB is there-
fore:

$$V_{AB}= (I\rho/2\pi)[(1/r_{AM} - 1/r_{AN}) - (1/r_{BM} - 1/r_{BN})] \qquad (9.54)$$

in which the symbol V, by convention, denotes the voltage drop between two
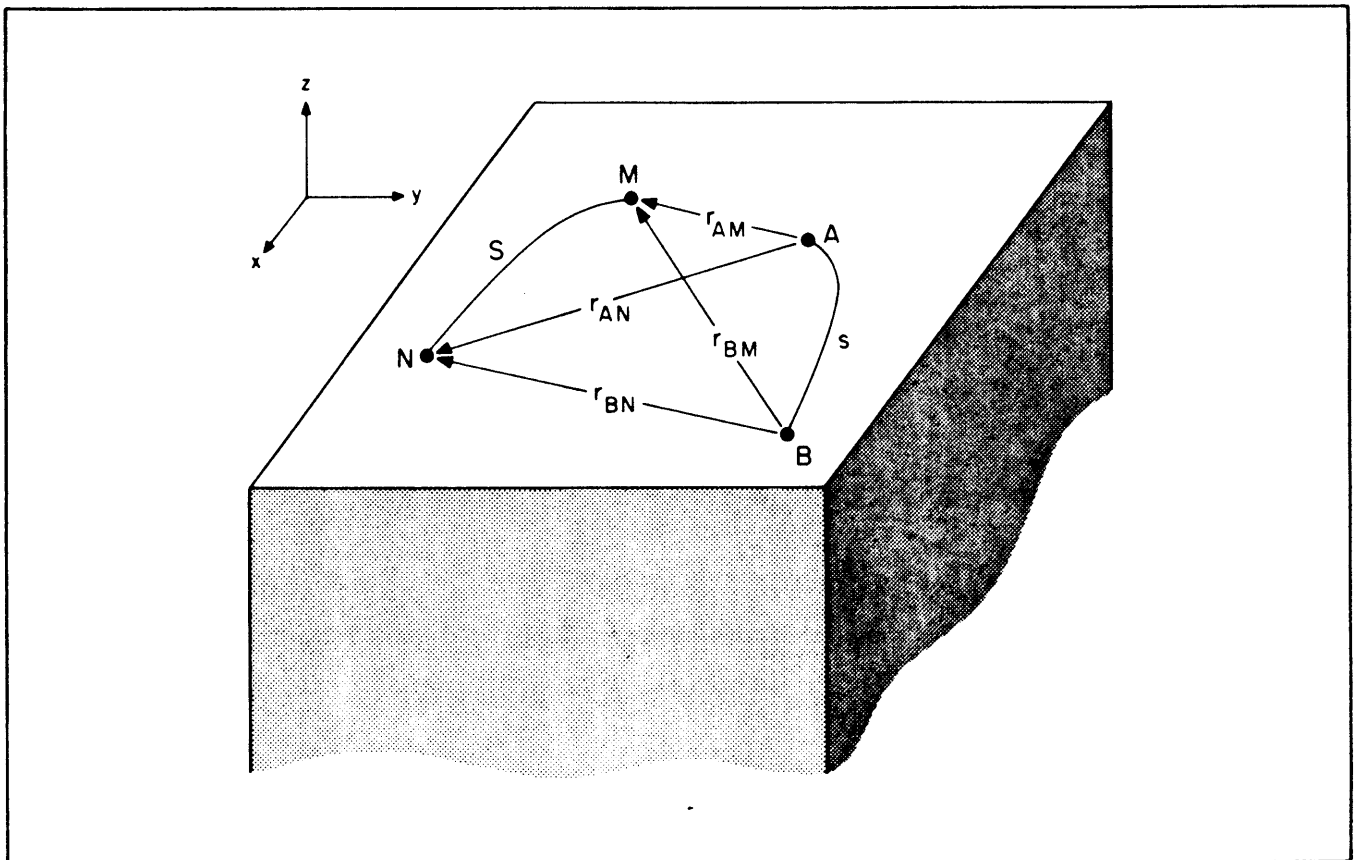specified points.

Figure 9.3. Arbitrary four-electrode array on the surface of a homogeneous earth.

Of particular interest in this volume is the voltage drop for a collinear dipole-dipole array, depicted in Figure 9.4. In this array, both the transmitter and receiver dipoles have a length "a", and are separated by "na". Therefore we have: AM = a + na, AN = na, BM = a + a + na, and BN = a + na. Equation (9.54) then reduces to

$$V_{AB} = (1\rho/2\pi)[1/(n+1)a - 1/na - 1/(n+2)a + 1/(n+1)a]$$

which in turn reduces to (dropping the subscript for V):

$$V = I\rho/[\pi an (n+1) (n+2)] \qquad (9.55)$$

Equation (9.55) is used to calculate the ground resistivity for the dipole-dipole array. Note that this relation has the form of Ohm's Law, relating resistance (R) to voltage and current:

$$V = IR$$

This is the potential analog of equation (9.15). The term in brackets in equation (9.55) is called the "geometric factor", since it describes the locations of the electrode grounding points.
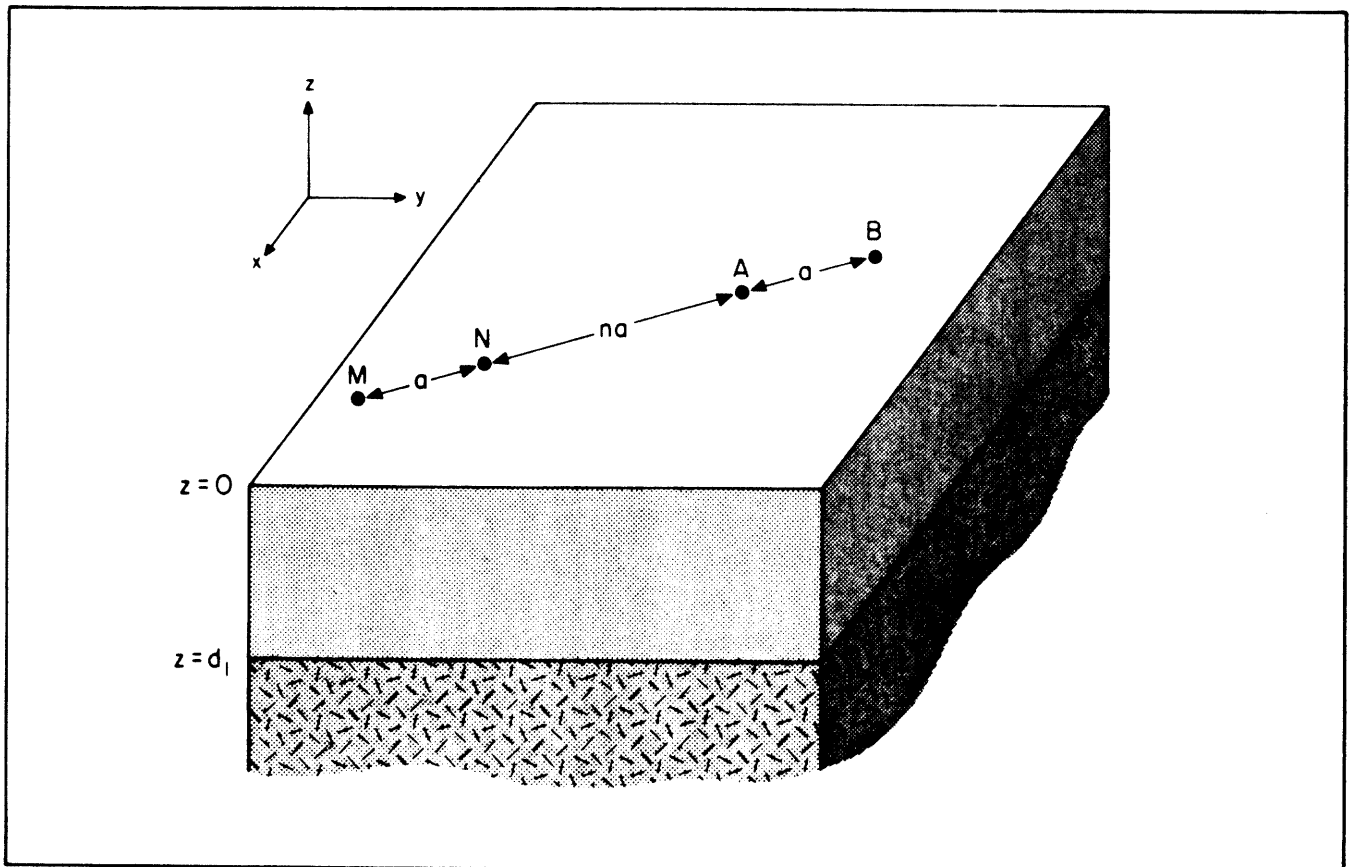

Figure 9.4. Collinear dipole-dipole array on the surface of a two-layered earth.

**Two-Layered
Earth
Resistivity**

It is useful for purposes of computer modeling to determine the response of a two-layered earth with resistivities ρ_1 and ρ_2, as illustrated in Figure 9.4. We seek a potential ϕ_1 in the first layer, which is composed of a primary part (the source) and a secondary part (due to layering):

$$\phi_1 = \phi_P + \phi_{S1} \tag{9.56}$$

The primary potential is

$$\phi_P = (I\rho_1/2\pi)(z^2 + r^2)^{-1/2} \tag{9.57}$$

in which z is the depth in the ground. This relation can be written in terms of a zeroth-order Bessel function of the first kind:

$$\phi_P = (I\rho_1/2\pi)\int_0^\infty J_0(\lambda r)e^{-\lambda z}d\lambda \tag{9.58}$$

in which λ is the integration variable. The secondary potential, after application of appropriate boundary conditions and some algebra, can be written

$$\phi_{S1} = (I\rho_1/2\pi)\int_0^\infty \left[-2u_{12}e^{-2\lambda d_1}/(1+u_{12}e^{-2\lambda d_1})\right]J_0(\lambda r)d\lambda \tag{9.59}$$

in which

$$u_{12} = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \tag{9.60}$$

The potential difference between electrodes A and B at the surface of the earth is

$$V_1 = (\phi_{AM} + \phi_{AN})_1 - (\phi_{BM} + \phi_{BN})_1 \tag{9.61}$$

in which the primary and secondary potentials are contained implicitly in the ϕ terms. For the dipole-dipole array, these are:

$$\phi_{AM} = (I\rho_1/2\pi)[1/(n+1)a + 2\sum_{k=1}^\infty (-u_{12})^k/[(n+1)^2a^2 + (2kd_1)^2]^{1/2}] \tag{9.62}$$

$$\phi_{AN} = (I\rho_1/2\pi)[1/na + 2\sum_{k=1}^\infty (-u_{12})^k/[(na)^2 + (2kd_1)^2]^{1/2}] \tag{9.63}$$

$$\phi_{BM} = (I\rho_1/2\pi)[1/(n+2)a + 2\sum_{k=1}^\infty (-u_{12})^k/[(n+2)^2a^2 + (2kd_1)^2]^{1/2}] \tag{9.64}$$

$$\phi_{BN} = (I\rho_1/2\pi)[1/(n+1)a + 2\sum_{k=1}^\infty (-u_{12})^k/[(n+2)^2a^2 + (2kd_1)^2]^{1/2}] \tag{9.65}$$

The results of these calculations are used for simple resistivity modeling in induced polarization work. They lead to a series of curves in which resistivity may be plotted as a function of n-spacing for various depths to the layer interface and for various resistivity contrasts between the two layers. The field curves are "curve-matched" to the theoretical curves in order to establish the layering parameters. A similar procedure may be used for a three-layered earth, but the algebra is more complicated, and the number of theoretical curves needed for the process is rather unmanageable.

9.7
AC SOLUTIONS TO THE VECTOR WAVE EQUATION

Harmonic Time-Dependence

We will now examine the response of a non-polarizable, homogeneous half-space to an alternating current. The purpose here is to develop an equation for frequency-dependent resistivity, or complex impedance, so that frequency-dependent data collected in the field can be analyzed for the separate effects of direct current flow ("galvanic" processes) and indirect or induced current flow ("electromagnetic coupling" processes).

The following treatment assumes a harmonic time-dependency for the electric field,

$$\vec{E} = \vec{E}_o \, e^{-i\omega t} \tag{9.66}$$

in which $\vec{E}_o$ is the peak field strength, $\vec{E}$ is the instantaneous field strength at time t, and ω is the angular frequency ($\omega = 2\pi f$, where f is the linear AC frequency). We recall that

$$e^{-i\omega t} = \cos \omega t - i\sin \omega t \tag{9.67}$$

Maxwell's equations are sufficiently general that a harmonic time-dependency can be assumed implicitly.

Let us consider a harmonic, time-varying field in a source-free region. From equations (9.25), (9.43), (9.26), (9.44), (9.40), and (9.47), we have

$$\nabla^2 \vec{A} + k^2 \vec{A} = 0 \tag{9.68}$$

$$\nabla^2 \vec{A}^* + k^2 \vec{A}^* = 0 \tag{9.69}$$

$$\nabla^2 \phi + k^2 \phi = 0 \tag{9.70}$$

$$\nabla^2 \phi^* + k^2 \phi^* = 0 \tag{9.71}$$

$$\nabla^2 \vec{\pi} + k^2 \vec{\pi} = 0 \tag{9.72}$$

$$\nabla^2 \vec{\pi}^* + k^2 \vec{\pi}^* = 0 \tag{9.73}$$

in which k is the so-called propagation constant, or wave number, the reciprocal of the radian wavelength. It is given by

$$k^2 = -\mu\epsilon\omega^2 + i\mu\sigma\omega \tag{9.74}$$

The first term ($\mu\epsilon\omega^2$) is the displacement term, and the second ($i\mu\sigma\omega$) is the conduction term. The complex form of the propagation constant can be written

$$k = \alpha + i\beta \tag{9.75}$$

The real part, α, is the phase constant, given by

$$\alpha = \omega \left[(\mu\epsilon/2) \left(\sqrt{1 + (\sigma/\omega\epsilon)^2} + 1 \right) \right]^{1/2} \tag{9.76}$$

The imaginary part, β, is the attenuation factor, given by

$$\beta = \omega \left[(\mu\epsilon/2) \left(\sqrt{1 + (\sigma/\omega\epsilon)^2} - 1 \right) \right]^{1/2} \tag{9.77}$$

Frequency-Dependence of k

Most earth materials are relatively conductive, and the conduction term tends to dominate the displacement term in equation (9.74), providing that the frequency is low (i.e., around 1 Hz). However, the two terms become more comparable at higher frequencies, making it important to understand their relative contributions as a function of frequency and media properties. In order to explicitly evaluate frequency-dependency, let us write

$$k^2 = \mu\omega \left(-\omega\epsilon + i\sigma \right) \tag{9.78}$$

and compare the contribution of σ with respect to $\omega\epsilon$ by finding the frequency at which they are equal:

$$f' = \omega/2\pi = \sigma/2\pi\epsilon \qquad (\sigma = \omega\epsilon) \tag{9.79}$$

Keller and Frischknecht (1966) pose two opposite examples for comparison: a shale, for which $\epsilon = 10^{-6}$ fd/m and $\rho = 10$ ohm-m, and a granite, for which $\epsilon = 1.6 \times 10^{-10}$ fd/m and $\rho = 10^5$ ohm-m. In the case of the shale, we obtain $f' = 16$ kHz. In the case of the granite, we obtain a slightly lower $f' = 10$ kHz. Therefore, we have $\sigma \gg \omega\epsilon$ for the normal frequency range (0.01 to 100 Hz) used for induced polarization type surveys. Note, however, that the $\sigma \gg \omega\epsilon$ constraint does not hold for downhole electric and induction logs, which are normally obtained at frequencies of 1-20 kHz. These logs may be dominated by displacement effects, while surface induced polarization surveys are dominated by conduction effects. Hence, one must be very careful in comparing data from these two frequency ranges.

SPECIAL CASE: $\sigma \gg \omega\epsilon$

If we assume that conduction effects predominate, $\sigma \gg \omega\epsilon$, we have

$$\alpha = \beta = \sqrt{\mu\sigma\omega/2} \tag{9.80}$$

The propagation constant is therefore

$$k = \alpha + i\beta = (1 + i)\sqrt{\mu\sigma\omega/2} \tag{9.81}$$

For a downward-traveling plane wave, the harmonically-dependent electric field E_x can be written:

$$E_x = E_o \, e^{-kz} \tag{9.82}$$

Substituting for k,

$$E_x = E_o \, e^{-z\sqrt{\mu\sigma\omega/2}} \, e^{-iz\sqrt{\mu\sigma\omega/2}} \tag{9.83}$$

This can also be written

$$E_x = E_o \, e^{-z/\delta} \, e^{-iz/\delta} \tag{9.84}$$

with the definition

$$\delta \equiv 1/\alpha = \sqrt{2/\mu\sigma\omega} \tag{9.85}$$

The term δ is the "skin depth", a term which describes the effective depth of penetration of a plane wave through a conductive material. Note that at $z = \delta$,

$$E_x = E_o \, e^{-1} \tag{9.86}$$

and the incident plane wave is attenuated by a factor of $1/e$. Thus, "skin depth" is the distance over which a plane wave is attenuated by a factor of $1/e$. If we assume a magnetic permeability of 1.256×10^{-6} hr/m, as is the case for most earth materials, we can write equation (9.85) in more practical terms:

$$\delta = 503 \sqrt{\rho/f} \qquad \text{(meters)} \tag{9.87}$$

This important result shows that, in a plane wave field, penetration is highest at high resistivities and low frequencies. This has direct application to controlled source audiofrequency magnetotelluric (CSAMT) measurements, which are sometimes used for detailing alteration patterns in the sediments above hydrocarbons.

The wavelength (λ) of the harmonic signal is

$$\lambda = 2\pi\delta \tag{9.88}$$

The phase or propagation velocity is

$$v_P = \omega/\alpha = \sqrt{2\omega/\mu\sigma} \tag{9.89}$$

SPECIAL CASE: $\sigma \ll \omega\epsilon$

When displacement currents dominate conductive currents, $\sigma \ll \omega\epsilon$, the phase constant becomes

$$\alpha = \omega \sqrt{\mu\epsilon} \tag{9.90}$$

and the attenuation constant becomes zero. The skin depth in such an infinitely resistive medium is infinite. The propagation velocity is

$$v_P = \omega/\alpha = 1/\sqrt{\mu\epsilon} \tag{9.91}$$

Note that in a vacuum, the propagation velocity is

$$v_P = 1/\sqrt{\mu_o \epsilon_o} \tag{9.92}$$

which is equal to the speed of light.

9.8 COMPLEX IMPEDANCE OF THE ELECTRIC FIELD

Development of the Relations

The purpose of this section is to develop the basic relations which describe the complex impedance of the ground. Consider two wires S and s lying on the surface of a homogeneous earth (Figure 9.3). We recall that the electric Hertz vector

has a primary part $\vec{\pi}_p$ due to the source current, and a secondary part $\vec{\pi}_s$ due to the transient field induced by the source. The primary potential can be written in terms of the inhomogeneous Helmholtz equation, which is the general form of equation (9.72):

$$(\nabla^2 + k^2)\,\vec{\pi}_p = -\vec{P}_v/\epsilon \tag{9.93}$$

The solution to this is

$$|\vec{\pi}_p| = -(i\omega\mu_o\,Ids/4\pi k^2)\int_o^\infty (1/r)\,e^{-ikr}\,dr \tag{9.94}$$

It is convenient to express this in terms of the zeroth-order Bessel function:

$$|\vec{\pi}_p| = -(i\omega\mu_o\,Ids/4\pi^2)\int_o^\infty (\lambda/u_o)\,e^{-u_o z}\,J_o(\lambda r)d\lambda \tag{9.95}$$

The symbol u_o is used in place of $\sqrt{\lambda^2 - k_o{}^2}$ as a matter of convenience; in general,

$$u_i = \sqrt{\lambda^2 - k_i{}^2} \tag{9.96}$$

Note that equation (9.95) is the complex analog of the DC equation (9.59) developed earlier for a two-layered earth.

The secondary potential $\vec{\pi}_s$ satisfies the homogeneous Helmholtz equation with no sources present:

$$(\nabla^2 + k^2)\,\vec{\pi}_s = 0 \tag{9.97}$$

Solutions in the x-y plane are of the form

$$|\vec{\pi}_s| = F(\lambda)e^{\pm u_i z}\,J_n(\lambda r) \tag{9.98}$$

After the application of boundary conditions, we can write for π_x and π_z the relations:

$$\pi_x = -(i\omega\mu_o\,Ids/4\pi k_o{}^2)\int_o^\infty [(\lambda/u_o)\,e^{u_o z} + (\lambda/u_o)e^{-u_o z}\,R_{TE}(\lambda)]\,J_o(\lambda r)d\lambda \tag{9.99}$$

$$\pi_z = -(i\omega\mu_o\,Ids/4\pi k_o{}^2)\frac{\partial}{\partial x}\int_o^\infty [(1/\lambda)\,e^{u_o z}R_{TE}(\lambda) + (1/\lambda)e^{u_o z}\,R_{TM}]\,J_o(\lambda r)d\lambda \tag{9.100}$$

The media properties have been recombined in terms of the transverse electric mode reflection coefficient, $R_{TE}(\lambda)$, and the transverse magnetic coefficient, $R_{TM}(\lambda)$, which are given by

$$R_{TE}(\lambda) = (N_o - Y_1)/(N_o + Y_1) \tag{9.101}$$

$$R_{TM}(\lambda) = (I_o - Z_1)/(I_o + Z_1) \tag{9.102}$$

in which we have defined:

$$I_o = u_o/i\omega\epsilon_o \tag{9.103}$$

$$N_o = u_o/i\omega\mu_o \tag{9.104}$$

$$Z_j = I_j\left[Z_{j+1} + I_j\tanh(u_j h_j)\right]/\left[I_j + Z_{j+1}\tanh(u_j h_j)\right] \tag{9.105}$$

$$Z_n = I_n = u_n/(\sigma_n + i\omega\epsilon_n) \tag{9.106}$$

$$Y_j = N_j\left[Y_{j+1} + N_j\tanh(u_j h_j)\right]/\left[N_j + Y_{j+1}\tanh(u_j h_j)\right] \tag{9.107}$$

$$Y_n = N_n = u_n/(\sigma_n + i\omega\epsilon_n) \tag{9.108}$$

The electric and magnetic fields can be obtained from equations (9.48) and (9.49), which may be re-written as

$$\vec{E} = k_o^2 \, \vec{\pi} + \vec{\nabla} \, \vec{\nabla} \cdot \vec{\pi} \tag{9.109}$$

$$\vec{H} = (\sigma_o + i\omega\epsilon_o) \, \vec{\nabla} \times \vec{\pi} \tag{9.110}$$

With substitutions of equations (9.99) and 9.100), the x-component of the electric field can be written as

$$E_x = k_o^2 \, \pi_x + \partial/\partial x \, (\partial\pi_x/\partial x + \partial\pi_z/\partial z) \, |_{z=0}^{\infty} \tag{9.111}$$

which is converted to Bessel function form:

$$E_x = - (i\omega\mu_o \, Ids/4\pi k_o^2) \left\{ k_o^2 \int_o^\infty [(\lambda/u_o)(1 + R_{TE}(\lambda))] \, J_o(\lambda r) d\lambda \right.$$
$$\left. + (\partial^2/\partial x^2) \int_o^\infty [(\lambda/u_o)(1 + R_{TE}(\lambda)) - (u_o/\lambda)(R_{TE}(\lambda) + R_{TM}(\lambda))] \, J_o(\lambda r) d\lambda \right\} \tag{9.112}$$

For ease in notation, we can write this as

$$E_x = Ids \, [- P(r) + \partial^2 Q(r)/\partial x^2] \tag{9.113}$$

in which we define

$$P(r) = - (i\omega\mu_o/4\pi) \int_o^\infty [(\lambda/u_o)(1 + R_{TE}(\lambda))] \, J_o(\lambda r) d\lambda \tag{9.114}$$

$$Q(r) = - (i\omega\mu_o/4\pi k_o^2) \int_o^\infty [(\lambda/u_o)(1 + R_{TE}(\lambda)) - (u_o/\lambda)(R_{TE}(\lambda) + R_{TM}(\lambda))] \, J_o(\lambda r) d\lambda \tag{9.115}$$

Finally, the impedance Z_{Ss} between the two wires lying in the x-y plane can be written:

$$Z_{Ss} = \int_M^N \int_A^B [P(r) \cos \epsilon + \partial^2 Q(r)/\partial S \, \partial s] \, dS \, ds \tag{9.116}$$

in which ϵ is the angle between the straight line element MN and the element AB. The voltage drop between A and B is then

$$V_{AB} (\omega) = I_{MN} \, Z_{Ss} (\omega) \tag{9.117}$$

Functional Behavior of P and Q

We can think of the complex impedance as being the sum of two functions, P and Q:

$$Z_{Ss} = P + Q \tag{9.118}$$

with

$$P = \int_M^N \int_A^B P(r) \cos \epsilon \tag{9.119}$$

$$Q = \int_M^N \int_A^B (\partial^2 Q(r)/\partial S \, \partial s) \, dS \, ds \tag{9.120}$$

The "P function" is called the inductive function because it is purely inductive in nature, as suggested by the cos ϵ dependence. It is instructive to examine the behavior of the P function under a homogeneous earth assumption. In such a

case, equation (9.114) reduces to:

$$P(r) = -(\omega\mu_o/4\pi)\int_0^\infty [2\lambda/(u_o + u_1)] \, J_o(\lambda r)d\lambda \qquad (9.121)$$

At low frequencies, the quasi-static approximation ($k_o \rightarrow 0$, $u_o \rightarrow \lambda$) applies, leading to:

$$P(r) = -(i\omega\mu_o/2\pi r)[1 - (1 + ik_1 r)e^{-ik_1 r}]/k_1^2 r^2$$

$$= (i\omega\mu_o/2\pi k_1^2)[(1/r^2)e^{-ik_1 r} + (1/r^3)(e^{-ik_1 r} - 1)] \qquad (9.122)$$

The P function is related to dipole size, frequency, and earth resistivity by

$$P \propto a^2 f/\rho \qquad (9.123)$$

Therefore, we can conclude that the P function is maximized when the following conditions hold:

1. Dipole sizes are large
2. Frequency is high
3. Resistivity is low
4. Wires are parallel (because $\cos \epsilon \rightarrow 1$)

It can also be shown that P is maximized by high-over-low resistivity layering. The P function is minimized when wires are perpendicular.

The "Q" function is known as the grounding function, since it is specifically dependent upon the dipole grounding points. Taking the limit of the homogeneous earth case, we have

$$Q(r) = -(i\omega\mu_o/2\pi)\int_0^\infty \left[\lambda/(k_1^2 u_o + k_o^2 u_1)\right] J_o(\lambda r)d\lambda \qquad (9.124)$$

With the quasi-static approximation ($k_o \rightarrow 0$, $u_o \rightarrow \lambda$),

$$Q(r) = -(i\omega\mu_o/2\pi)\int_0^\infty k_1^{-2} \, J_o(\lambda r)d\lambda \qquad (9.125)$$

$$= -i\omega\mu_o/2\pi k_1^2 r$$

If we neglect displacement currents, we have $k_1^2 = i\mu\sigma\omega$, and

$$Q(r) = \rho/2\pi r \qquad (9.126)$$

which is frequency-independent. Without displacement currents, the inductive P term is zero at DC, so $Z_{ss} = Q = Q(r)$, in which case

$$V_{AB} = I\rho/2\pi r \qquad (9.127)$$

This is identical to the result obtained earlier in equation (9.53).

In the case of a homogeneous earth, the Q function is constant and is proportional to the resistivity of the earth.

In the case of a layered earth, Q becomes dependent upon the same factors which govern the P function:

$$Q \propto a^2 f/\rho \qquad (9.128)$$

Hence, Q is maximized when:

1. Dipole sizes are large
2. Frequency is high
3. Resistivity is low

The Q function is also dependent upon the depth to the layering interfaces and the resistivity contrasts involved.

Figure 9.5 illustrates the relative contributions of P and Q to the overall coupling curve for a two-layered earth. Note that although P may dominate Q in terms of overall magnitude of effect, the more interesting changes are demonstrated by the Q function. The behavior of Q is discussed in more detail by Wynn (1974) and by Wynn and Zonge (1975).
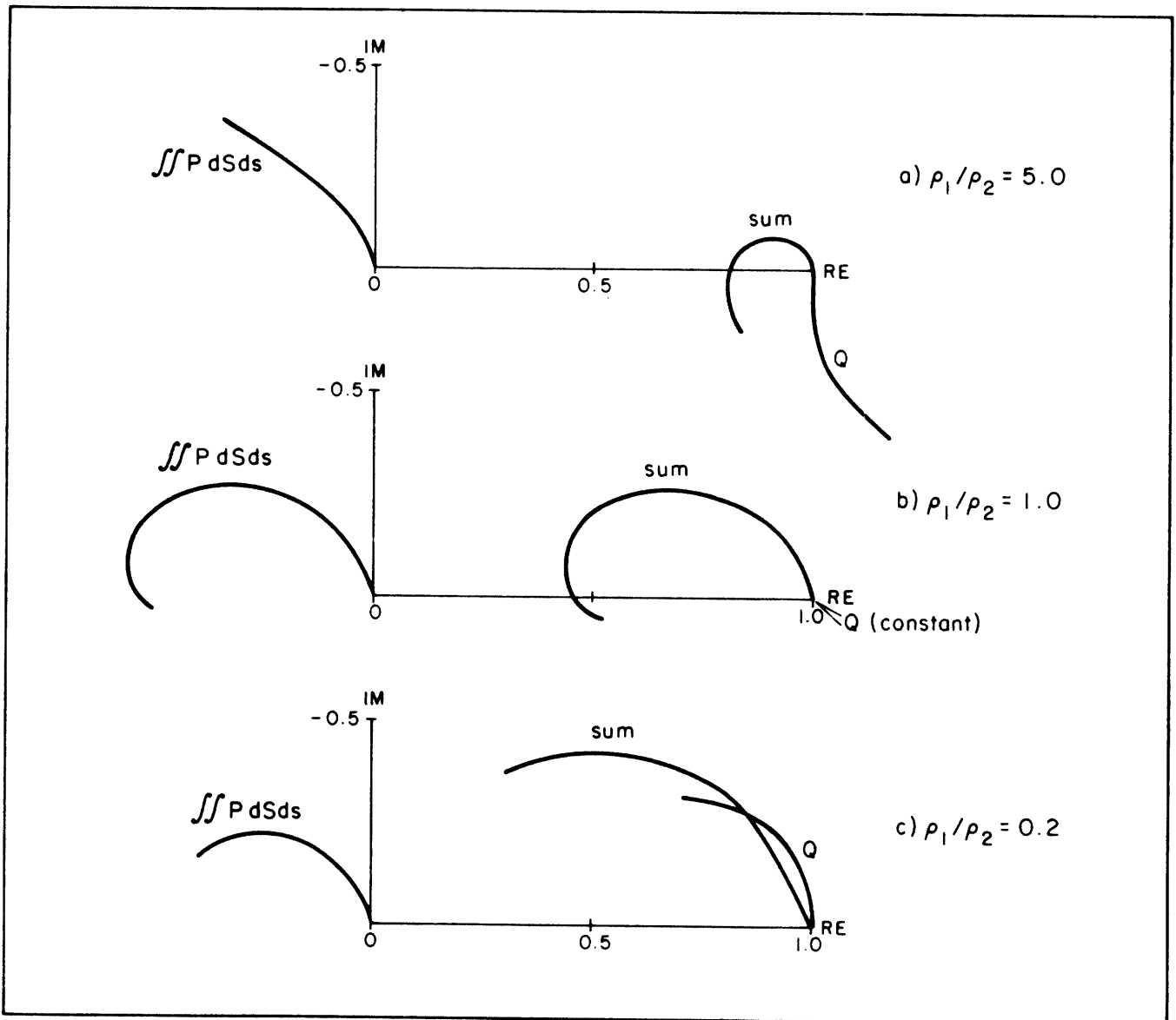

Figure 9.5. Comparison of the P and Q functions for a two-layered earth with three different resistivity contrasts. The depth to the layer interface is the same in all three drawings (d/a = 1.0). Resistivity of the top layer is ρ_1, resistivity of the bottom layer is ρ_2. The curves are plotted in the complex plane, as described in Chapter 1. After Wynn and Zonge (1975).

Separation of Polarization and Inductive Coupling

Complex resistivity field data consist of the combined effects of ground capacitance and inductive coupling. In order to utilize each of these two parameters, it is necessary to separate them in a process called "decoupling." This is an extremely difficult problem, and the exact solution of the complex impedance equation (9.116) is a practical impossibility in all but the simplest geologic environments. The approach in dealing with coupling has historically followed three patterns:

1) Ignore or minimize coupling
2) Quadratic extrapolation
3) Iterative curve-matching or dispersion techniques

The contribution due to coupling can be minimized by using very small dipoles or very low frequencies. None of these are practical in normal petroleum application, however, since large dipoles and multiple frequencies are often needed for this kind of work. In addition, by minimizing the effects of coupling, one also minimizes its potential use in data interpretation, since the frequency dependence of the induced polarization parameter and the REM parameter in general would be lost. Consequently, minimizing coupling is not an acceptable approach to the problem.

The quadratic extrapolation technique, first described by Hallof (1974), is a better alternative. The technique involves a quadratic extrapolation of phase data from three low frequencies to DC (where the electromagnetic coupling effect is zero). A continuous function of the following form is assumed:

$$f(x_1) = \phi_1 = Ax_1^3 + Bx_1^2 + Cx_1 + D$$
$$f(x_2) = \phi_2 = Ax_2^3 + Bx_2^2 + Cx_2 + D$$
$$f(x_3) = \phi_3 = Ax_3^3 + Bx_3^2 + Cx_3 + D \tag{9.129}$$
$$f(x_4) = \phi_4 = Ax_4^3 + Bx_4^2 + Cx_4 + D$$

The four equations are solved simultaneously in order to obtain the extrapolated phase angle ϕ_c from the phase angles ϕ_f at frequencies 1, 2, 3, and 4. If only three equations are used, the equations simplify to a parabolic quadratic:

$$f(x_1) = \phi_1 = Ax_1^2 + Bx_1 + C$$
$$f(x_2) = \phi_2 = Ax_2^2 + Bx_2 + C \tag{9.130}$$
$$f(x_3) = \phi_3 = Ax_3^2 + Bx_3 + C$$

For binary-related frequencies (e.g., 0.125, 0.25, 0.5, 1.0, . . . Hz), the three-point equation is:

$$\phi_c = (8/3)\phi_1 - 2\phi_2 + (1/3)\phi_3 \tag{9.131}$$

For harmonically-related frequencies of the type used in complex resistivity surveys (1, 3, 5, 7, . . . times the fundamental frequency; e.g., 0.125, 0.375, 0.625, 0.875, . . . Hz), the equation is:

$$\phi_c = (15/8)\phi_1 - (5/4)\phi_3 + (3/8)\phi_5 \tag{9.132}$$

Three-point extrapolation is a useful technique in low-coupling, low-polarization environments. However, the technique tends to severely undercorrect or overcorrect in high-coupling oilfield environments, as explained in Chapter 1. In addition, the assumption of a polarization response which does not vary with fre-

quency is unrealistic, and it ignores the wealth of available spectral information which is inherent in a multifrequency survey. As a result, three-point extrapolation is useful as a first-order processing method, but detailed work is best done with more exact decoupling methods.

There exist two more direct methods of dealing with the coupling problem. The first method involves an iterative curve-matching approach developed by Zonge (Wynn and Zonge, 1975). This technique is described in Chapter 1. A second method involves the fitting of dispersion relations of the type described by Cole and Cole (1941), and inverting the data. This method has largely been developed by Pelton (Pelton et al., 1978), and is used by Phoenix Geophysics in petroleum and mining applications.

To date, the main purpose of decoupling has been to remove the coupling effect from the field data, leaving a pure polarization parameter for interpretation. However, direct decoupling techniques have provided a whole new set of electromagnetic information which can be used in petroleum exploration, as outlined by Zonge, Van Reed, and Young (1980) and Hughes et al. (1983), and as pointed out in the case histories in this volume. It is hoped that this type of information will be more fully exploited as multifrequency electrical methods gain wider acceptance in the petroleum industry.

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